P P SAVANI UNIVERSITY

First Semester of B. Tech. Examination
January 2022
SESH1070 Fundamentals of Mathematics

24.01.2022, Monday   Time: 09:00 a.m. To 11:30 a.m.   Maximum Marks: 60

Instructions:
1. The question paper comprises of two sections.
2. Section I and II must be attempted in same answer sheet.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I
Answer the Following: (Attempt any Five)

Q - 1	Examine if Rolle's theorem if applicable to the function $f(x)$, where $f(x) = x, 0 \leq x < 1$ and $f(1) = 0$.	[06]
Q - 2	Find 'c' of the Lagrange's mean value theorem for the function $f(x) = x(x-1)(x-2)$, $x \in \left[0, \frac{1}{2}\right]$.	[06]
Q - 3	Verify the Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in $[a, b]$.	[06]
Q - 4	Check whether $f(x) \begin{cases} 1, & x < 0 \\ -1, & x > 0 \end{cases}$ is continuous or discontinuous. If discontinuous then discuss the type of discontinuity.	[06]
Q - 5	Examine for convergence of the series $\frac{x^2}{2\sqrt{1}} + \frac{x^3}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \cdots$	[06]
Q - 6	Examine for convergence of the series $\frac{1}{3\cdot7} + \frac{1}{4\cdot9} + \frac{1}{5\cdot11} + \cdots$	[06]
Q - 7	Show that the given series is divergent $\frac{1}{5} + \frac{\sqrt{2}}{7} + \frac{\sqrt{3}}{9} + \frac{\sqrt{4}}{11} + \cdots$	[06]
Q - 8	Examine for convergence the series whose n^{th} term is $\left[\left(\frac{n+1}{n}\right)^{n+1} - \left(\frac{n+1}{n}\right)\right]^{-n}$	[06]

SECTION – II
Answer the Following: (Attempt any Five)

Q - 1	Using Taylor's series to find the expansion of $\log_e x$ in powers of $(x - 1)$. Hence find the value of log 1.1 correct up to four decimal places.	[06]
Q - 2	Determine the Maclaurin's series for $\cos 4t$ as far as the term in t^6.	[06]
Q - 3	Evaluate $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$	[06]
Q - 4	Evaluate $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$	[06]
Q - 5	Find the inverse of $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$ using Gauss-Jordan method.	[06]
Q - 6	Solve the following system of linear equations by Gauss-Jordan Method: $x + 2y + z = 8, 2x + 3y + 4z = 20, 4x + 3y + 2z = 16$	[06]
Q - 7	Show that the matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$ is diagonalizable. Also, find the modal and diagonal matrices.	[06]
Q - 8	Using Cayley-Hamilton theorem, find A^2, A^{-1} and A^{-2} from $A = \begin{bmatrix} -2 & 1 \\ 2 & 4 \end{bmatrix}$	[06]

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